WEEKLY TEST OYJ - TEST - 16
SOLUTION Date 04-08-2019

[PHYSICS]

1. Given $E = \dfrac{q}{4\pi\varepsilon_0 x^2}$. Hence the magnitude of the electric intensity at a distance $2x$ from charge q is

$$E' = \frac{q}{4\pi\varepsilon_0 (2x)^2} = \frac{q}{4\pi\varepsilon_0 x^2} \times \frac{1}{4} = \frac{E}{4}$$

Therefore, the force experienced by a similar charge q at a distance $2x$ is

$$F = qE' = \frac{qE}{4}$$

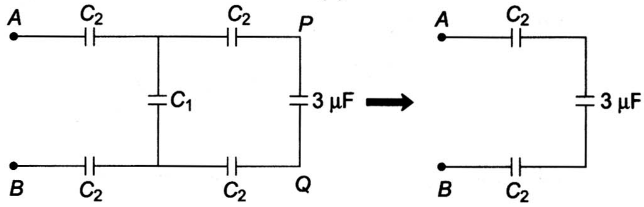
Hence the correct choice is (d).

2. The system will be in equilibrium if the net force on charge q at one vertex due to charges q at the other two vertices is equal and opposite to the force due to charge Q at the centroid, i.e. (here a is the side of the triangle)

$$\frac{\sqrt{3}\, q^2}{4\pi\varepsilon_0 a^2} = -\frac{Qq}{4\pi\varepsilon_0 \left(\dfrac{a}{\sqrt{3}}\right)^2}$$

which gives $Q = -\dfrac{q}{\sqrt{3}}$. Hence the correct choice is (b).

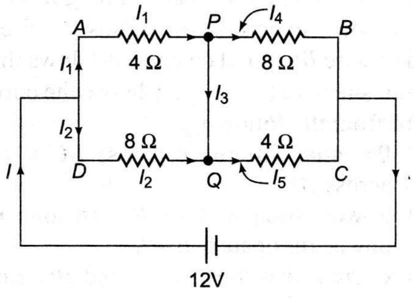
3. The network reduces to that shown in Fig. 21.42. The correct choice is (a).



4. Refer to Fig. 22.81. The equivalent resistance is

$$R = \frac{12 \times 12}{(12+12)} = 6\ \Omega$$

$$\therefore \quad I = \frac{12\ \text{V}}{6\Omega} = 2\ \text{A}$$



From Kirchhoff's junction rule,

$$I_1 + I_2 = I \qquad (1)$$

Using Kirchhoff's loop rule to loop *APQDA*, we have

$$4I_1 - 8I_2 = 0 \quad \Rightarrow I_1 = 2I_2 \qquad (2)$$

Equations (1) and (2) give $I_1 = \frac{4}{3}$ A and $I_2 = \frac{2}{3}$ A.

Similarly $I_4 = \frac{2}{3}$ A and $I_5 = \frac{4}{3}$ A.

Applying junction rule at *P*,

$$I_1 = I_3 + I_4$$

$$\therefore \quad I_3 = I_1 - I_4 = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}\ \text{A}$$

The positive sign shows that current I_3 flows from *P* to *Q*. Hence the correct choice is (c).

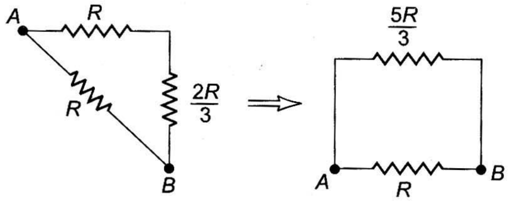
5. $I_1 = \frac{12\ \text{V}}{4\Omega} = 3$ A

Applying Kirchhoff's loop rule to loop *ABCDE,*

$$2I_2 + E - 4I_1 = 0$$

Putting $I_1 = 3$ A and $I_2 = 0$, we get $E = 12$ V

6. The circuit shown in Fig. can be redrawn as shown in Fig.



$\therefore \quad R_{AB} = \frac{5R}{8}$, which is choice (c).

7. The correct choice is (b).

8. The emfs of cells connected in reverse polarity cancel each other. Hence cells marked 2, 3 and 4 together cancel the effect of cells marked 5, 6 and 7 and the circuit reduces to that shown in Fig. 22.91. Now cells 1 and 8 are in reverse polarity. Hence the voltmeter reading = 5 – 5 = 0 V. Hence the correct choice is (d).

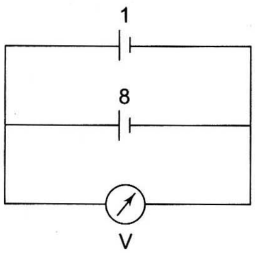


Fig. 22.91

9. The two sub circuits are closed loops. They cannot send any current through the 3 Ω resistor. Hence the potential difference across the 3 Ω resistor is zero, which is choice (a).

10. $R = \dfrac{\rho l}{\pi r^2}$. Since the two wires are made of the same material, resistivity ρ is the same for wires AB and BC. Since the wires have equal lengths, it follows that $R \propto 1/r^2$. Hence

$$\frac{R_{AB}}{R_{BC}} = \frac{1}{4}, \text{ i.e } R_{BC} = 4R_{AB}$$

Since the current, is the same in the two wires, it follows from Ohm's law ($V = IR$) that $V_{BC} = 4\ V_{AB}$. Hence choice (a) is wrong. Now power dissipated is $P = I^2 R$. Since I is the same, $P \propto R$. Hence

$$\frac{P_{BC}}{P_{AB}} = \frac{R_{AB}}{R_{BC}} = 4$$

Hence chioce (b) is correct. Choice (c) is wrong because current density (i.e. current per unit area) is different in wires AB and BC because their cross-sectional areas are different. The electric field in a wire is $E = V/l$. Since the two wires have the same length (l), E is proportional to potential difference (V). Since $V_{BC} = 4\ V_{AB}$, $E_{BC} = 4E_{AB}$. Hence choice (d) is also incorrect.

11. When the two heaters are connected in parallel, the resistance of the combination is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Now $$\frac{1}{t_1} = \frac{V^2}{QR_1} \text{ and } \frac{1}{t_2} = \frac{V^2}{QR_2}$$

Also $$\frac{1}{t} = \frac{V^2}{Q}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{1}{t_1} + \frac{1}{t_2}$$

or $$t = \frac{t_1 t_2}{(t_1 + t_2)}$$

Hence the correct choice is (c).

12. Let R be the value of each resistance. The resistances of combinations I, II, III and IV are $3R$, $R/3$, $2R/3$ and $3R/2$ respectively. Now, power dissipation is inversely proportional to resistance. Hence the correct choice is (b).

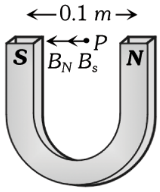
13. (c) Net magnetic field at mid point $P, B = B_N + B_S$

where $B_N =$ magnetic field due to N- pole

$B_S =$ magnetic field due to S- pole

$B_N = B_S = \frac{\mu_0}{4\pi}\frac{m}{r^2}$

$= 10^{-7} \times \frac{0.01}{\left(\frac{0.1}{2}\right)^2} = 4 \times 10^{-7} T$

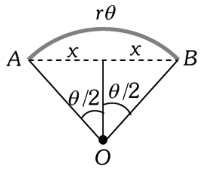


$\therefore B_{net} = 8 \times 10^{-7} T.$

14. (d) From figure

$\sin\frac{\theta}{2} = \frac{x}{r}$

$\Rightarrow x = r\sin\frac{\theta}{2}$



$r\theta = l$

$r = \frac{l}{\theta}$

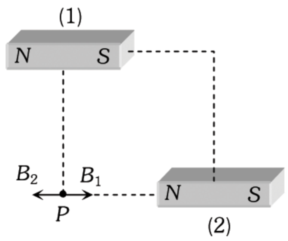
Hence new magnetic moment $M' = m(2x) = m.2r\sin\frac{\theta}{2}$

$= m.\frac{2l}{\theta}\sin\frac{\theta}{2} = \frac{2ml\sin\theta/2}{\theta} = \frac{2M\sin(\pi/6)}{\pi/3} = \frac{3M}{\pi}$

15. (d) Due to wood moment of inertia of the system becomes twice but there is no change in magnetic moment of the system.

Hence by using $T = 2\pi\sqrt{\frac{I}{MB_H}} \Rightarrow T \propto \sqrt{I} \Rightarrow T' = \sqrt{2}\,T$

16. (a) Point P lies on equatorial line of magnet (1) and axial line of magnet (2) as shown



$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{M}{d^3} = 10^{-7} \times \frac{1000}{(0.1)^3} = 0.1\,T$

$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2M}{d^3} = 10^{-7} \times \frac{2 \times 1000}{(0.1)^3} = 0.2\,T$

$\therefore B_{net} = B_2 - B_1 = 0.1\,T$

17. (a) Both points A and B lying on the axis of the magnet and on axial position

$$B \propto \frac{1}{d^3} \Rightarrow \frac{B_A}{B_B} = \left(\frac{d_B}{d_A}\right)^3 = \left(\frac{48}{24}\right)^3 = \frac{8}{1}$$

18. (a) $M = mL = 4 \times 10 \times 10^{-2} = 0.4 A \times m^2$

19. (d) Magnetic potential at a distance d from the bar magnet on it's axial line is given by

$$V = \frac{\mu_0}{4\pi} \cdot \frac{M}{d^2} \Rightarrow V \propto M \Rightarrow \frac{V_1}{V_2} = \frac{M_1}{M_2}$$

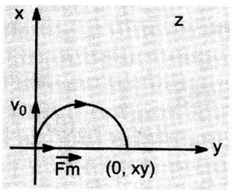
$$\Rightarrow \frac{V}{V_2} = \frac{M}{M/4} \Rightarrow V_2 = \frac{V}{4}$$

20. **(d)** $R_1 < R_2$

and $R = \frac{mv}{qB}$ of $\left(\frac{m}{q}\right)_1 < \left(\frac{m}{q}\right)_2$

21. A

22. **(c)** $y = 2r = \frac{2mv_0}{B_0 q} = \frac{2v_0}{B_0 \alpha}$



Here, $\frac{q}{m} = \alpha$

23. **(b)**

24.
(b) e.m.f. induced across the rod PQ is

$$\mathcal{E} = \vec{\mathbf{B}} \cdot (\vec{\mathbf{l}} \times \vec{\mathbf{v}})$$
$$= Blv \sin\theta$$
$$= 2 \times 2 \times 2 \times \sin 30$$
$$\mathcal{E} = 4\text{V}$$

Free electrons of the rod shift towards right due to force $q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$

Thus end P is at higher potential

or $V_P - V_Q = 4\text{V}$

25. **(d)** Potential difference across capacitor

$$V = Bvl = \text{constant}$$

Therefore, change stored in the capacitor is also constant. Thus, current through the capacitor is zero.

26. C

27. **(c)** When the rod rotates, there will be an induced current in the rod. The given situation can be treated as if a rod A of length $3l$ is rotating in clockwise direction, while another rod B of length $2l$ is rotating in the anticlockwise direction with the same angular speed ω

As $\quad \mathcal{E} = \frac{1}{2}B\omega l^2$

For A : $\quad \mathcal{E}_A = \frac{1}{2}B\omega(3l)^2$

and $\quad \mathcal{E}_B = \frac{1}{2}B(-\omega)(2l)^2$

Resultant induced e.m.f. will be :

$$\mathcal{E} = \mathcal{E}_A + \mathcal{E}_B = \frac{1}{2}B\omega l^2(9-4)$$

$$\mathcal{E} = \frac{5}{2}B\omega l^2$$

28. D

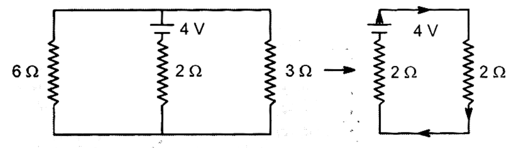
29. **(c)** Current in the YY' direction is from Y' to Y but the current is constant and hence the magnetic flux through the coil is constant. Therefore the current in the coil is zero.

30. **(c)** Motional e.m.f.

$$\mathcal{E} = Bvl$$

$$\mathcal{E} = (2)(2)(1) = 4V$$

This acts as a cell of e.m.f. $\mathcal{E} = 4V$ and internal resistance $r = 2\Omega$. The simple circuit can be drawn as follows :



$\therefore$ Current through the connector

$$I = \frac{4}{2+2} = 1A$$

Magnetic force on connector

$$F_m = IlB$$
$$= (1)(1)(2)$$
$$= 2N \quad \text{(towards left)}$$

Therefore, to keep the connector moving with a constant velocity, a force of 2N will have to be applied towards right.

[CHEMISTRY]

31. (a) $d = \frac{n \times \text{at. wt.}}{a^3 \times \text{Av. No.}}$ or $2.165 = \frac{n \times 58.5}{(562 \times 10^{-10})^3 \times 6 \times 10^{23}}$

$\therefore$ $n = 4$ rank of unit cell

$\therefore$ AB has fcc structure,

$\therefore$ $d_{A^+-B^-} = \frac{a}{2}$

32. (b) No. of atoms in bcc = 2

No. of atoms in fcc = 4

$\therefore$ Ratio $= \frac{2}{4} = 0.5$

33. (a)

$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{(2r)^2 + (2r)^2}$$
$$r + r + 2R = 2\sqrt{2}r$$

$\therefore$ $2R = 2(\sqrt{2}r - r)$

$R = (\sqrt{2} - 1)r$, $\frac{R}{r} = 0.41$, $\therefore$ $\frac{r}{R} = 2.41$

34. (a) $r_+ + r_- = \frac{a}{2}$ for fcc

35. (d) $r = K[\ ]^n$ and $K = Ae^{-E_a/RT}$. $E_a = 0$ for free radical combination. K is constant with T.

36. (c) $K = Ae^{-E_a/RT}$. K depends upon E_a, T and nature of reaction; but always increases with T. Thus rate always increases with T.

37. (a) Temperature coefficients are : I. $\frac{K_1}{K_2} = \frac{E_{a_1}}{2.303\,R}\frac{[T_2 - T_1]}{T_1 T_2} = 2$

II. $\frac{K_1'}{K_2'} = \frac{E_{a_2}}{2.303\,R}\left[\frac{T_2 - T_1}{T_1 T_2}\right] = 3$

if $\frac{K_1}{K_2} < \frac{K_1'}{K_2'}$ then $E_{a_2} > E_{a_1}$

38. (d) $r = K[N_2O_5]$ I order as unit of $K = \text{sec}^{-1}$

$$2.40 \times 10^{-5} = 3.0 \times 10^{-5}[N_2O_5]$$

$$\therefore \quad [N_2O_5] = \frac{2.40}{3.0} = 0.8\,M$$

39. **(a) follow rate law**

40. (b) In photo initiated primary process rate of reaction is directly proportional to intensity of light uses.

41. (a) $\frac{P^\circ - P_s}{P^\circ} = \frac{n}{n+N}(1+\alpha)$ ΔP is minimum since $\alpha = 0$ for urea. Thus, urea solution will have its V.P. closer to solvent.

42. (b)

$$\pi_{\text{urea}} = \pi_{\text{NaCl}}$$

$$\frac{w}{m} \times \frac{ST}{V} = \frac{w}{m} \times \frac{ST}{V} \times (1+\alpha)$$

$$\frac{6}{60} \times \frac{1000}{100} \times ST = \frac{w \times 1000 \times ST}{58.5 \times 100} \times 2 \qquad (\because \alpha = 1 \text{ for NaCl})$$

$\therefore$ $w = 2.925$ $\quad\therefore$% by wt. vol. = 2.925%

43. (a) Only liquid freezes at freezing point. Thus equilibrium between solid and liquid forms of solvent exist at freezing point.

44. (c)

$$\pi_{Na_2SO_4} = \pi_{\text{Glucose}}$$

$$CRT\,(1 + 2\alpha) = CRT$$

$$0.004\,(1 + 2\alpha) = 0.01$$

$$\therefore \quad \alpha = 0.75 \text{ or } = 75\%$$

45. **(a) fact**

46. (b) Current flows from anode to cathode in external circuit of electrolytic cell and thus electrons flow from anode to cathode through external wires.

47. (a)

$$E^\circ_{\text{cell}} = E^\circ_{OPFe} + E^\circ_{RPH_2O} = 0.44 + 1\ 23 = 1.67\ \text{V}$$

$$\therefore \quad \Delta G^\circ = -nE^\circ F = -2 \times 1.67 \times 96500\ \text{J} = -322.31\ \text{kJ mol}^{-1}$$

48. (c) The salt bridge possesses the electrolyte having nearly same ionic mobilities of its cation and anion.

49. (b)

$$\frac{w}{E} = \frac{i \cdot t}{96500}$$

$$0.01 \times 2 = \frac{10 \times 10^{-3} \times t}{96500}, \qquad t = 19.3 \times 10^4 \text{ sec}$$

50. (d)

$$F = N \times e, \qquad 96500 = 6.023 \times 10^{23} \times e$$

$\therefore$ $e = 1.602 \times 10^{-19}$

51. (a) Lower is the critical temperature of gas, more are van der Waals' forces of attractions among gaseous molecules, more is adsorption.

52. (c) Negative catalyst is adsorbed on the surface of catalyst to make it inert.

53. (d) Alkaline hydrolysis of ester also called saponification is irreversible

54. (b) Randomness decreases during adsorption.

55. (d) Lyophilic sols usually organic, self stabilizing because there sols are reversible and highly hydrated in solution.

56. (a)

$$Cr^{3+} + 3e \longrightarrow Cr; \qquad -\Delta G_1^\circ = 3 \times 0.74 \times F$$

$$Cr^{3+} + e \longrightarrow Cr^{2+}; \qquad -\Delta G_2^\circ = 1 \times 0.40 \times F$$

$$- \qquad - \qquad +$$

$$Cr^{2+} + 2e \longrightarrow Cr; \quad -\Delta G_3^\circ = 2 \times E^\circ \times F = (3 \times 0.74 - 1 \times 0.40)\,F = 1.82\,F$$

$\therefore$ $E^\circ = 0.91$ V

57. (c) $(Mn^{+8/3})_3 \longrightarrow 3Mn^{6+} + 10e$

$\therefore$ 10 Faraday charge is required for conversion of 1 mole of Mn_3O_4 to MnO_4^{2-}.

58. (c) As_2S_3 is negative sol. Higher is the valence of effective ion (*i.e.*, positive ion) more is coagulating power.

59. (d) Cleansing action is due to micellisation and emulsifying action.

60. A

[MATHEMATICS]

61. Given, $g(x) = 1 + \sqrt{x}$ and $f\{g(x)\} = 3 + 2\sqrt{x} + x$ ...(i)

$\Rightarrow$ $f(1 + \sqrt{x}) = 3 + 2\sqrt{x} + x$

Put $1 + \sqrt{x} = y \Rightarrow x = (y - 1)^2$

$\therefore$ $f(y) = 3 + 2(y - 1) + (y - 1)^2 = 2 + y^2$

$\therefore$ $f(x) = 2 + x^2$

62. $\because g(x) = 1 + x - [x]$ (put $x = n \in Z$)

and $g(x) = 1 + n + k - n = 1 + k$ (put $x = n + k$)

(where, $n \in Z$, $0 < k < 1$)

Now, $$f\{g(x)\} = \begin{cases} -1, & g(x) < 0 \\ 0, & g(x) = 0 \\ 1, & g(x) > 0 \end{cases}$$

Clearly, $g(x) > 0, \forall x$

So, $f\{g(x)\} = 1, \forall x$

63. Since, $f(x) = 2[x] + \cos x = \begin{cases} \cos x, & 0 \le x < 1 \\ 2 + \cos x, & 1 \le x < 2 \\ 4 + \cos x, & 2 \le x < 3 \end{cases}$

Since, $\cos x < 1$ and $2 + \cos x > 1$.

So, $f(x)$ never gives the value one. Hence, $f(x)$is into.

If $0 < \alpha < \pi - 3$, then $f(\pi - \alpha) = f(\pi + \alpha)$

So, $f(x)$ is not one-one.

64. Given, $n(B) = 21, n(H) = 26, n(F) = 29,$

$$n(H \cap B) = 14, n(H \cap F) = 15, n(F \cap B) = 12$$

and $n(B \cap H \cap F) = 8$

$$\therefore \; n(B \cup H \cup F) = n(B) + n(H) + n(F) - n(B \cap H) - n(H \cap F) - n(B \cap F) + n(B \cap H \cap F)$$

$$= 21 + 26 + 29 - 14 - 15 - 12 + 8 = 43$$

65. Here, we see that every element of codomain there exist a pre-image, hence it is onto.

66. $x + \frac{1}{x} = 2 \Rightarrow \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = 0$

$$\Rightarrow \quad \frac{x-1}{\sqrt{x}} = 0 \Rightarrow x = 1$$

So, the principal value of $\sin^{-1} x$ is $\frac{\pi}{2}$.

67. Given, $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$

$$\Rightarrow \quad \sin^{-1} x = \sin^{-1} y = \sin^{-1} z = \frac{\pi}{2}$$

$$\Rightarrow \quad x = y = z = 1$$

$$\therefore \; \sum \frac{(x^{101} + y^{101})(x^{202} + y^{202})}{(x^{303} + y^{303})(x^{404} + y^{404})} = \sum \frac{(1+1)(1+1)}{(1+1)(1+1)}$$

$$= \sum 1 = 3$$

68. $$\tan^{-1}\left[\frac{\frac{x-1}{x+1} + \frac{2x-1}{2x+1}}{1 - \left(\frac{x-1}{x+1}\right)\left(\frac{2x-1}{2x+1}\right)}\right] = \tan^{-1}\left(\frac{23}{36}\right)$$

$$\Rightarrow \frac{2x^2 - 1}{3x} = \frac{23}{36} \Rightarrow 24x^2 - 12 - 23x = 0 \Rightarrow x = \frac{4}{3}, -\frac{3}{8}$$

But x cannot be negative.

69. Let $I = (\sec^{-1} x)^2 + (\text{cosec}^{-1} x)^2$

$$= (\sec^{-1}x + \text{cosec}^{-1}x)^2 - 2\sec^{-1}x\,\text{cosec}^{-1}x$$

$$= \frac{\pi^2}{4} - 2\sec^{-1}x\left(\frac{\pi}{2} - \sec^{-1}x\right)$$

$$= \frac{\pi^2}{4} + 2\left[(\sec^{-1}x)^2 - \frac{\pi}{2}(\sec^{-1}x) + \frac{\pi^2}{16} - \frac{\pi^2}{16}\right]$$

$$= \frac{\pi^2}{8} + 2\left[\left(\sec^{-1}x - \frac{\pi}{4}\right)^2\right]$$

$$\therefore I_{max} = \frac{\pi^2}{8} + 2\left[\frac{9\pi^2}{16}\right] = \frac{5\pi^2}{4}$$

70. $$\sum_{r=1}^{n}\tan^{-1}\frac{1}{2r^2} = \sum_{r=1}^{n}[\tan^{-1}(2r+1) - \tan^{-1}(2r-1)]$$

$$= \tan^{-1}(2n+1) - \tan^{-1}1 = \tan^{-1}\left(\frac{n}{n+1}\right)$$

71. $$\left(\frac{\pi}{2} - \sin^{-1}x\right)^2 + (\sin^{-1}x)^2 = \frac{\pi^2}{4} + 2(\sin^{-1}x)^2 - \pi\sin^{-1}x$$

$$= \frac{\pi^2}{8} + 2\left[\sin^{-1}x - \frac{\pi}{4}\right]^2$$

Here, $m = \frac{\pi^2}{8}, M = \frac{5\pi^2}{4}$

$\therefore$ $\frac{M}{m} = 10$

72. Given, $\sqrt{1+\cos 2x} = \sqrt{2}\cos^{-1}(\cos x)$

$\therefore$ $\sqrt{2\cos^2 x} = \sqrt{2}\,x \Rightarrow \sqrt{2}\,|\cos x| = \sqrt{2}\,x$

For $x \in \left[\frac{\pi}{2}, \pi\right]$, $|\cos x| = -\cos x$

$$-\sqrt{2}\cos x = \sqrt{2}\,x \Rightarrow -\cos x = x$$

$\therefore$ $\cos x = -x$

Hence, no solution exist.

73. (c) $27^{\cos x} + 81^{\sin x} = 3^{3\cos x} + 3^{4\sin x}$

$$\geq 2\cdot\sqrt{3^{3\cos x}\cdot 3^{4\sin x}} \qquad (\because \text{AM} \geq \text{GM})$$

$$= 2\cdot 3^{(3\cos x + 4\sin x)/2}$$

$$\geq 2\cdot 3^{\frac{1}{2}(-5)} \qquad (\because -5 \leq 3\cos x + 4\sin x \leq 5)$$

$$= 2\cdot 3^{-\frac{5}{2}} = 2\cdot 3^{-2}\cdot 3^{-\frac{1}{2}} = \frac{2}{9\sqrt{3}}$$

74. Clearly $\left[-\frac{2x}{\pi}\right]+\frac{1}{2}=-\left(\left[\frac{2x}{\pi}\right]+\frac{1}{2}\right)$

$\Rightarrow$ f(x) is an odd function.

Hence (A) is the correct answer.

75. Given function is defined if ${}^{10}C_{x-1} > 3\ {}^{10}C_x$

$\Rightarrow \frac{1}{11-x} > \frac{3}{x} \Rightarrow 4x > 33$

$\Rightarrow x \geq 9$ but $x \leq 10 \Rightarrow x = 9, 10.$

Hence (D) is the correct answer.

76. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\sin[x]}{[x]} = \frac{\sin(-1)}{(-1)} = \sin 1$

and $\lim_{x\to 0^+} f(x) = 0$ as it is given that f(x) = 0 for [x]=0

So, $\lim_{x\to 0} f(x)$ doesn't exist.

Hence (D) is the correct answer

77. $\lim_{h\to o} [\tan^2 (0 + h)] = \lim_{h\to o} [\tan^2 (0-h)] = [\tan^2 0] = 0$

So, f(x) is continuous at x = 0.

Since f(x) = 0 in the neighbourhood of 0, f'(0) = 0.

Hence (B) is the correct answer.

78. Let $l = \lim_{x\to 0}\left(\frac{a^x + b^x + c^x}{3}\right)^{2/x}$ (form 1^∞)

$\log l = \lim_{x\to 0} \frac{2}{x}\log\left(\frac{a^x + b^x + c^x}{3}\right)$ [Using L'Hospital]

$\log l = \frac{2}{3}\lim_{x\to 0}\left(\frac{a^x \log a + b^x \log b + c^x \log c}{a^x + b^x + c^x}\right) = 2\frac{\log(abc)}{3}$

$\Rightarrow \log l = \log (abc)^{2/3}$

$\Rightarrow l = (abc)^{2/3}$

Hence (B) is the correct answer.

79 . $f(x) = a\left[\dfrac{\log\ (1+ax)}{ax}\right] + b\left[\dfrac{\log(1-bx)}{(-bx)}\right]$

So, $\lim\limits_{x\to 0} f(x) = a.\ 1 + b.1 = a + b\ = f(0)$ $\left[\lim\limits_{x\to 0} \dfrac{\log(1+x)}{x} = 1\right]$

Hence (B) is the correct answer.

Alternative Sol.

$= \lim\limits_{x\to 0} \dfrac{a - ab\ \ x + b + abx}{(1+ax)\ \ (1-bx)} = a + b$ (by L'Hospital's Rule)

So, f(0) = a + b, if f is continuous.

80 . $f(x) = \begin{cases} (x^2-1)(x-1)(x-2)+\cos x & x<1,\ \ x>2 \\ -(x^2-1)(x-1)(x-2)+\cos x & 1\le x\le 2 \end{cases}$

f(x) is differentiable every where possibly not at x = 1, 2.

After testing the condition of differentiability, we can see that f(x) is not differentiable at x = 2 .

Hence (D) is the correct answer.

81 . Put $\dfrac{1}{\sin^2 x} = t \ge 1$ so, that

LHS = $\lim\limits_{t\to\infty}\left(1^t + 2^t + \cdots + n^t\right)^{1/t} = \lim\limits_{t\to\infty} n\left[\left(\dfrac{1}{n}\right)^t + \left(\dfrac{2}{n}\right)^t + \cdots + 1\right]^{1/t}$

$= n[0 + 0 + ... + 1]^0 = n.$

Hence (D) is the correct answer.

82. LHS = $\lim\limits_{x\to\infty}\left(\dfrac{1\cdot 2\cdot 3\cdots n}{n^n}\right)^{1/n} \times \dfrac{1}{m} = \lim\limits_{x\to\infty}\dfrac{1}{m}\left[\left(\dfrac{1}{n}\right)\left(\dfrac{2}{n}\right)\left(\dfrac{3}{n}\right)\cdots\left(\dfrac{n-1}{n}\right)\left(\dfrac{n}{n}\right)^{1/n}\right]$ = S (say)

$\Rightarrow\ \ln S = \lim\limits_{x\to\infty}\left[\ln\left(\dfrac{1}{m}\right) + \dfrac{1}{n}\sum\limits_{r=1}^{n}\left(\dfrac{r}{n}\right)\right]$

$= -\ln m + \int\limits_0^1 \ln x\ dx = -\ln m + [x\ln x - x]_0^1$

$= -\ln m + (-1) = -\ln em = \ln\dfrac{1}{em} \Rightarrow\ S = \dfrac{1}{em}.$

Hence (A) is the correct answer.

83. $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{x\to 0}\frac{f(x)+f(h)-f(x)}{h} = \lim_{h\to 0}\frac{f(h)}{h} = \lim_{h\to 0}\frac{h^2g(x)}{h} = 0$.

Hence (D) is the correct answer.

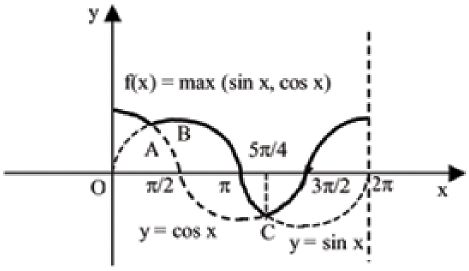
84. Required limit = $\lim_{x\to\infty}\frac{1}{n}\cdot\frac{(e^{1/n})^n-1}{e^{1/n}-1} = (e-1)\lim_{x\to\infty}\frac{1}{\left(\frac{e^{1/n}-1}{1/n}\right)}$

$= (e-1)\times 1 = e-1$.

Hence (C) is the correct answer.

85. Clearly A, B and C are the critical points.

Hence (C) is correct.

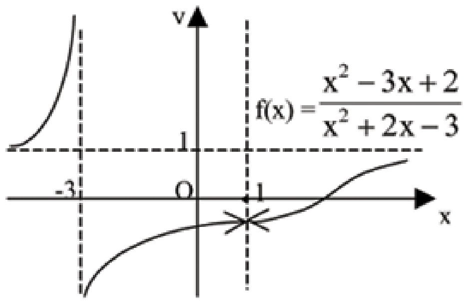


86. $f(x) = \frac{x^2-3x+2}{x^2+2x-3} = \frac{(x-1)(x-2)}{(x-1)(x+3)}$

$= \frac{x-2}{x+3},\quad x\neq 1, -3$

$\frac{df}{dx} = \frac{(x+3)-(x-2)}{(x+3)^2}$

$= \frac{5}{(x+3)^2} > 0\ \forall\ x \neq 1, -3.$



Clearly f (x) is increasing in its domain.

Hence (C) is correct.

87. f is continuous at '0' and f'(0-) > 0 and f' (0 +) < 0. Thus f has a local maximum at '0'. Hence (A) is correct.

88 . f(x) = 1- x +a, x< 1

= 2x+3, $x \geq 1$

Local minimum value of f(x) at x = 1, will be 5

i.e. 1- x + a ≥ 5 at x = 1 or, a ≥ 5. Hence (A) is correct.